

Contrast Enhancement by Nonlinear Diffusion Filtering

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Abstract — To enhance the visual quality of an image that is degraded by uneven light, an effective method is to estimate the illumination component and compress it. Some previous methods either have defects of halo artifacts or contrast loss in the enhanced image due to incorrect estimation. In this paper, we discuss this problem and propose a novel method to estimate the illumination. The illumination is obtained by iteratively solving a nonlinear diffusion equation. During the diffusion process, surround suppression is embedded in the conductance function to specially enhance the diffusive strength in textural areas of the image. The proposed estimation method has the following two merits: First, the boundary areas are preserved in the illumination, and thus, halo artifacts are prevented; and second, textural details are preserved in the reflectance to not suffer from illumination compression, which contributes to the contrast enhancement in the result. Experimental results show that the proposed algorithm achieves excellent performance in artifact removal and local contrast enhancement.

Index Terms —contrast; image enhancement; illumination estimation; nonlinear diffusion; halo artifacts;

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I. INTRODUCTION

The observed image is the multiplication of two components, namely, illumination and reflectance, which are described as intrinsic images in [1]. In a real scene, the illumination component is usually nonuniform and has a high dynamic range, whereas the reflectance reveals the details of the objects in the image. A digital acquisition device such as a consumer digital camera often suffers from varying light conditions because of its much narrower dynamic range than that of the illumination. The captured image could contain both underexposed and overexposed regions and has low local contrast in both areas. Therefore, an enhancement algorithm that focuses on addressing the illumination problem is required to enhance the image's visual quality.

There are several types of contrast enhancement algorithms that have been proposed, such as histogram equalization [2], [3] and perceptual enhancement [4], [5], among others. A large set of these algorithms are inspired by Retinex, which was originally a color constancy model that mimicked the color appearance of the human visual

system [6]. Algorithms of this type typically decompose the image into a low frequency component denoted as illumination and a high frequency component denoted as reflectance. The local contrast is enhanced by compressing the illumination or simply extracting the reflectance as the final result. Among the Retinex-inspired algorithms, a widely used version is the center/surround algorithm [7]–[13]. This type of algorithm estimates a smoothed version of the image and subtracts it from the original image to improve the contrast. The methods differ in the types of filters that are adopted to blur the image. The SSR (single-scale Retinex) [7] and MSR (multi-scale Retinex) [8] utilize a Gaussian filter to smooth the image. A modified method is the bilateral filter, whose weighting coefficients are a combination of spatial closeness and pixel value similarity [9], [10]. This method preserves the edges well in the illumination and avoids halo artifacts in the enhanced result. Recently, Wang designed a “bright-pass” filter to estimate the illumination, in which the weights were calculated by the frequency in the spatial domain of the relationship between the central pixel and its neighboring pixels [13]. Another set of Retinex-like enhancement algorithms is based on the variational framework [14]–[19]. These algorithms impose assumptions on both the reflectance and illumination, and they obtain both components by solving a quadratic optimization problem. The divergence of these models mainly involves different norms for the penalty terms and different constraints. The initial work is Kimmel's L_2 -based variation model [14], which uses sparsity terms on the illumination and reflectance as well as a fidelity term on the illumination. Ng *et al.* proposed a variational model that imposed a TV (total variation) term on the reflectance [15]. Recently, Fu *et al.* proposed a variational model for image enhancement, where the logarithmic transformation for the illumination and reflectance was abandoned to obtain a better measure of the edges [16]. The algorithm was efficient and preserved the edge details well.

To attenuate the influence of illumination unevenness and reveal the details in the underexposed and overexposed regions, the crucial process is to estimate the illumination and compress it. Because the estimation is an ill-posed problem, assumptions must be made to guide or constrain this process [20]. Different assumptions lead to illumination with different features and eventually affect the enhanced image. One of the assumptions is that the illumination component is spatially smooth, and thus, linear

filters are used for estimation [14]-[16]. This assumption usually causes halo artifacts around strong edges in the enhanced image because, in a real scene, the illumination is not always smooth and could change sharply in some areas [18]. In addressing halos, another type of assumption for illumination is described as “piecewise-smoothness”, which means that some discontinuities in the image should be preserved [9]-[11], [17], [19]. Considering this assumption, nonlinear convolutional filters are adopted to estimate the illumination. However, even though the nonlinear filters are effective in reducing the halo artifacts, they are not effective in contrast enhancement. This defect arises mainly because the nonlinear filters usually obtain small-scale smoothness and tend to over-preserve the various types of edges, including the texture edges. When illumination is compressed by gamma correction or simply removed in the subsequent process, this part of the texture information is compressed or lost in the enhanced result. Additionally, the definition of “piecewise-smoothness” for illumination is obscure, and what scale of smoothness should be obtained remains unclear. Therefore, it is necessary to make a more specific assumption and improve the method for illumination estimation as well.

Let us consider two types of edges in the image, namely, the boundary edges and texture edges. The boundary edges are considered to be strong contrast contours of objects or the border of two regions with different illumination conditions. This part of the edges should be preserved in the illumination because halo artifacts could easily occur. In contrast, texture edges consist in the interiors of regions and are important information for the visual quality of an image. They should be smoothed out from the illumination and transferred to the reflectance component for the final appearance. Overall, the illumination assumption made in this paper can be characterized as: flat as a constant in the textured areas and discontinuous in the boundary areas. It should be noted that precise illumination estimations of real scenes, which is the task in the computer vision area, is not involved in this paper. We make this estimation more concise for a practical contrast enhancement task.

Considering that the illumination estimation can be modeled as a selective smoothing process, it can be performed based on a more flexible framework that is described as nonlinear diffusion filtering. The nonlinear diffusion performs nonlinear filtering by solving a partial differential equation that models inhomogeneous heat or impurity transfer. The original equation was proposed by Perona and Malik to obtain an edge-preserving smoothing process and have an accurate scale-space interpretation for the images [21]. The early work on nonlinear diffusion mainly involves addressing its ill-posedness problem [22], [23] and developing different numerical schemes for solving the equation [25], [26]. Nonlinear diffusion has become an effective tool in the field of image smoothing [27], [28], image restoration [29], [30], image segmentation [31], and image enhancement [32].

In this paper, a novel contrast enhancement algorithm for images with uneven illumination is proposed. First, an illumination-reflectance decomposition is performed on the original image. The method used to estimate the illumination component is based on nonlinear diffusion

filtering. To better match the illumination estimate to the assumption made in this paper, a texture suppression mechanism is introduced to improve the performance of the filtering. Then, a logarithmic compression is conducted on the estimated illumination, which is followed by the final recombination of the reflectance and the compressed illumination to obtain the final result. Experiments show that the effectiveness of the algorithm is threefold: local contrast enhancement, global brightness promotion, and avoidance of halo artifacts.

The remainder of this paper is organized as follows. In Section II, the proposed nonlinear diffusion method for illumination estimation is described in detail, and the speed-up strategy is introduced. In Section III, the entire procedure of our contrast enhancement algorithm is presented. Analysis and comparison of several experimental results are presented in Section IV. Section V is the conclusion.

II. ILLUMINATION ESTIMATION BASED ON NONLINEAR DIFFUSION

The flexibility and effectiveness of nonlinear diffusion exist in that its smoothing performance can be tuned by changing the equation formation and determining a different set of parameters. For example, it can smooth undesired information while respecting region boundaries and other structures in the image, as long as some crucial parameters are set appropriately [33]. On the other hand, it tends to yield piecewise-constant-like images before it finally arrives at the globally constant solution [23]. The above features are potential merits for estimating the expected illumination in this paper. In this section, the nonlinear diffusion is first introduced. Then, the texture suppression used to improve the smoothing properties for better handling of the expected illumination is described. Finally, the entire nonlinear diffusion method for illumination estimation as well as the acceleration strategy is presented in detail.

A. Nonlinear Diffusion Equation

The original nonlinear diffusion equation has the following formula [21]:

$$\frac{\partial I(x, y)}{\partial t} = \text{div}\{g[|\nabla I(x, y)|] \cdot \nabla I(x, y)\} \quad (1)$$

with a Neumann boundary condition of

$$\partial_n I(x, y) = 0 \quad \text{on } \partial\Omega \quad (2)$$

Formula (1) denotes the evolution of image $I(x, y)$ per unit time. Here, $g(\cdot)$ is the conductance function that controls the diffusivity; it is set to be a monotonically decreasing function of an image gradient module $|\nabla I(x, y)|$ so that the diffusivities are strong in areas that have small gradients and weak in areas that have large gradients. As the evolution proceeds, the areas that have large discontinuities are preserved, whereas those that have small discontinuities such as plain areas or noise are smoothed out. Formula (2) indicates that there is no orthogonal diffusion to the image borders $\partial\Omega$. In theory, a globally constant image is the solution of the equation [23].

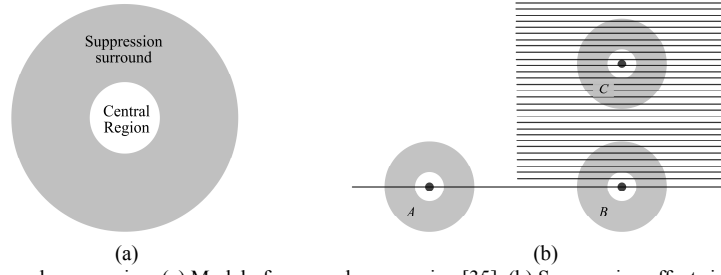


Fig. 1. Surround suppression. (a) Model of surround suppression [35]. (b) Suppression effects in different areas [36].

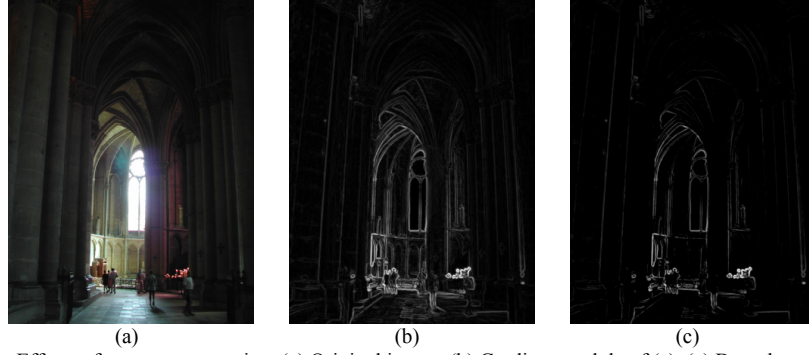


Fig. 2. Effects of texture suppression. (a) Original image. (b) Gradient module of (a). (c) Boundary template.

Although the original model of nonlinear diffusion yields an impressive result for selective smoothing, it is theoretically ill-posed [22], [23]. This problem occurs mainly because, in most cases, the variational function that is correlated with (1) is not convex [24]. Forward and backward diffusions simultaneously exist, which indicates that there are potentially multiple solutions to the equation. Fortunately, various regularization strategies in the research are sufficiently good to turn it well-posed for practical use [24].

With regard to the task of illumination estimation, directly applying equation (1) cannot effectively obtain the expected illumination. The reason is that its criterion for selective smoothing depends on the gradient module $|\nabla I(x, y)|$, which is unable to fully demarcate between texture edges and boundary edges. Some of the textures could have higher gradients than some boundaries and, hence, weaker diffusivities. This arrangement makes it difficult to guarantee both the smoothness of the textures and the retention of the boundaries, which are the required conditions for our expected illumination. Therefore, it is necessary to modify this smoothing criterion to gain a higher scale of smoothness in the texture edges than that in the boundary edges. As will be discussed next, surround suppression, which is a human visual property, is introduced to achieve this goal by suppressing the texture gradients.

B. Texture Suppression

Studies of neurophysiology have found that the human visual system has a mechanism to make prominent the contours of observed objects [34]. The main reason is that when recognizing objects by judging their contours, the human visual system has a surround suppression mechanism to suppress the disturbing texture information. This mechanism and its effects are shown in Fig. 1. Fig. 1(a) shows a model of complex receptive fields in the neurons of the visual cortex, where the surround suppression takes place [35]. It is basically divided into two areas: an annular

inhibitory surround called NCRF (non-classical receptive field) and a central region called CRF (classical receptive field). The stimulus sensed by NCRF always weakens the CRF's response to the stimulus in the central region. In the synthesized image Fig. 1(b), the sensations of the central points in region A, B, and C are suppressed by their surrounds [36]. The effect is strong in texture region B because its surround is full of textures, whereas the boundary regions A and C are less affected. Thus, the boundaries in A and C are highlighted.

Some of the researchers have applied the surround suppression mechanism to contour detection [35], [36]. In this application, the suppression process was embedded in the Canny edge detection operator to emphasize the detection of contour edges of objects. We extract some critical steps of the method in the literature [35] to suppress the texture gradients for our diffusion equation. The extracted suppression procedure for the gradient module $|\nabla I(x, y)|$ of an image is expressed as follows:

1. First, the suppression surround is simulated by constructing a suppression weighting function. Let $DoG_\sigma(x, y)$ be the following difference between two Gaussian functions with the scale σ :

$$DoG_\sigma(x, y) = \frac{1}{2\pi(4\sigma)^2} \exp\left(-\frac{x^2 + y^2}{2(4\sigma)^2}\right) - \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right) \quad (3)$$

The negative values in $DoG_\sigma(x, y)$ are set to 0, and normalization is performed to obtain the weighting function $w_\sigma(x, y)$:

$$w_\sigma(x, y) = \frac{H[DoG_\sigma(x, y)]}{\text{sum}(H[DoG_\sigma(x, y)])} \quad (4)$$

where

$$H[z] = \begin{cases} 0, & z < 0 \\ z, & z \geq 0 \end{cases} \quad (5)$$

The operator $\text{sum}(\cdot)$ in (4) denotes a summation.

The region with positive values in $w_\sigma(x, y)$ approximates an annular suppression surround.

2. Then, the suppression term $t(x, y)$ for each pixel is calculated by convolving the gradient module with the weighting function $w_\sigma(x, y)$:

$$t(x, y) = |\nabla I(x, y)| * w_\sigma(x, y) \quad (6)$$

In practice, the above convolution can be performed as a multiplication in the Fourier domain to reduce the computational complexity.

3. Finally, the suppressed gradient value $B(x, y)$ is obtained by subtracting the suppression term from the gradient module $|\nabla I(x, y)|$:

$$B(x, y) = H[|\nabla I(x, y)| - \alpha \cdot t(x, y)] \quad (7)$$

The suppression mechanism takes effect in such a way that the suppression terms $t(x, y)$ are small for the boundary edges and large for the texture edges. Here, α is the suppression strength factor, which directly influences the suppression effect. The suppressed gradient $B(x, y)$ can successfully assign high values in the boundary areas and small values in the texture areas. $B(x, y)$ is regarded as the boundary template in this paper.

Fig. 2 shows an example of the suppression effect. Fig. 2(b) is the gradient module of Fig. 2(a). As seen in Fig. 2(c), the boundary template has fewer textures, and the boundaries gain saliency.

C. Proposed Method for Illumination Estimation

From introducing the texture suppression in the previous subsection, the proposed nonlinear diffusion equation for illumination estimation is now given as

$$\begin{cases} \frac{\partial L(x, y, t)}{\partial t} = \text{div}\{g[B(x, y)] \cdot \nabla L(x, y)\} \\ L(x, y, 0) = S(x, y) \end{cases} \quad (8)$$

where $L(x, y)$ is the illumination image. The boundary template $B(x, y)$ is computed by performing suppression on the illumination gradient $|\nabla L(x, y)|$ using the method given in Section II-B. It replaces the former gradient module to be the new criterion for selective smoothing. The initial image for the iterations is set as the original degraded image $S(x, y)$.

Now that $B(x, y)$ can differentiate the texture edges and boundary edges, the conductance function $g(\cdot)$ in equation (8) is required to have a threshold property. It should be large where $B(x, y)$ is small, and vice versa. This approach will separate the diffusivities between the boundary areas and non-boundary areas. We refer to the analysis in [33] and slightly modify Tukey's biweight function to be the conductance function:

$$g(x) = \begin{cases} 0.92 \cdot \left[1 - \left(\frac{x}{K}\right)^2\right]^2 + 0.08 & x \leq K \\ 0.08 & x > K \end{cases} \quad (9)$$

where K is the threshold that differentiates the boundary edges from the suppressed texture edges. To achieve consistency in the whole diffusion process, K should be set as an adaptive threshold. Our tests show that when K is

set to be equal to the 90% - 95% value of the cumulative distribution function of $B^k(x, y)$, the estimated illumination is correct. The small value 0.08 is added to prevent an absolute diffusion shutoff, which causes unnaturalness around some of the slowly varying boundaries.

Considering that the reflectance is always below 1, during the diffusion process, the constraint $L > S$ (i.e., the illumination is greater than the luminance) should be imposed at each iteration. As the iterations proceed, the illumination estimate becomes the upper envelope of the image luminance.

The nonlinear diffusion equation (8) can be solved via a numerical scheme that has suitable discretization. One of the explicit schemes is expressed as follows [25]:

$$L_s^{k+1} = L_s^k + \tau \cdot \sum_{p \in N_e(s)} \frac{g_s^k + g_p^k}{2} \cdot (L_p^k - L_s^k) \quad (10)$$

where

$$g_i^k = g[B_i^k] \quad (11)$$

In (11), B_i^k denotes the boundary template at pixel i at the k th iteration. It gives the diffusivity g_i^k via the conductance function. In (10), L_s^{k+1} and L_s^k denote the illumination values at pixel s at the $k+1$ th and k th iterations, respectively. $N_e(s)$ denotes the 4 neighboring pixels that are connected to pixel s . τ is the discretized time step, which is limited to being below 0.25 for stability. For an image of size $m \times n$, the scheme (10) can be rewritten in a matrix-vector notation [26]:

$$L^{k+1} = L^k + \tau \cdot A(L^k) \cdot L^k \quad (12)$$

In this formation, L^{k+1} and L^k are $mn \times 1$ column vectors. $A(L^k)$ is a diffusive matrix of size $mn \times mn$, with its elements $a_{s,p}(L^k)$ expressed as

$$a_{s,p}(L^k) = \begin{cases} (g_s^k + g_p^k)/2, & p \in N_e(s) \\ - \sum_{p \in N_e(s)} (g_s^k + g_p^k)/2, & p = s \\ 0, & \text{elsewhere} \end{cases} \quad (13)$$

Last but not least, it is necessary to settle the number of iterations k to gain smoothness that is consistent with the illumination assumption, namely, flatness inside regions and discontinuity across region boundaries. Theoretically, a small k would not be able to smooth away the textures in the illumination and would cause contrast loss in the enhanced result. However, as k increases, the illumination becomes closer to a constant solution with vanished boundary areas, and the final image could remain with a lack of enhancement. Therefore, it is reasonable to stop the diffusion when only the texture areas are wiped out. We set a stopping criterion as

$$\frac{\|w_t \cdot (L^{k+1} - L^k)\|}{\|w_t \cdot L^k\|} \leq 7 \times 10^{-6} \quad (14)$$

where $\|\cdot\|$ denotes the L_2 norm operator. Here, w_t is a weight function described as

$$w_t = \frac{t^1(x, y)}{\max [t^1(x, y)]} \quad (15)$$

Here, $t^1(x, y)$ is the suppression term (6) at the first iteration. As described in Section II-B, $t^1(x, y)$ is especially large in the texture regions, and therefore, the weight w_t strengthens those areas. As a result, the stopping criterion (14) emphasizes the textural difference between the illuminations at the k th and $k+1$ th iterations. At the early stage of diffusion, the value on the left side of (14) declines very steeply as the diffusion gives priority to texture blurring. Then, it becomes stable at a fixed level because textures are already smoothed and only boundaries are merging slowly.

To summarize the discussion above, let S be the original image, and let L be the illumination image to be estimated. Then, the procedure of the proposed method is given as

Step1: Initialize $L^0 = S$.

Step2: Calculate the gradient module of L^k and perform texture suppression introduced in Section II-B to obtain the boundary template B^k .

Step3: Substitute B^k into $g(\cdot)$ and bring forth L^{k+1} using the explicit scheme as in (12).

Step4: Apply the constraint $L^{k+1} = \max(S, L^{k+1})$.

Step5: If L^{k+1} meets the stopping criterion (14), then execute Step 6. Otherwise, repeat step2 to step5.

Step6: $L = L^{k+1}$. The estimation is completed.

D. Acceleration for Estimation

Directly applying the explicit scheme in (12) for the diffusion process suffers from an extremely large number of iterations because of the strictly limited time step τ . Two methods are available to accelerate the estimation: the AOS (Additive Operator Splitting) scheme [26], to increase the update amount of each iteration, and the multi-resolution technique, to reduce the data complexity.

1) AOS Scheme

Unlike the explicit scheme in (12), a semi-implicit scheme has the following equation:

$$L^{k+1} = L^k + \tau \cdot A(L^k) \cdot L^{k+1} \quad (16)$$

The difference is that in (16), the data at the $k+1$ th iteration consists in both sides of the equation. Then, (16) requires solving a linear system, which gives implicit equations. The stability of such a scheme is guaranteed for an arbitrary time step, and thus, a larger time step can be set to speed up the iteration update [26]. Ultimately, the general semi-implicit scheme can be synthesized as

$$L^{k+1} = (E - \tau \cdot A(L^k))^{-1} \cdot L^k \quad (17)$$

where E is the $mn \times mn$ unit matrix.

Because the general semi-implicit scheme requires a large-scale matrix computation, its split versions are much more efficient. One of the split implementations is the AOS algorithm, which is generally expressed as

$$L^{k+1} = \frac{1}{2} \sum_{d=1}^2 (E - 2\tau \cdot A_d(L^k) \cdot L^k)^{-1} \cdot L^k \quad (18)$$

It can be seen that the iteration update is split into two processes. The AOS algorithm first performs 1-D diffusion in both the horizontal and vertical directions separately to obtain the two intermediate results. L^{k+1} is then obtained by taking the average of the two intermediate results. $A_d(L^k)$ denotes the general diffusive operator in the d th direction.

The main updating procedure of the AOS scheme for the estimation is concisely stated below; for more details, one can refer to research [26]. For an image of size $m \times n$, we have the following:

1. Horizontal diffusion.

The 1-D horizontal diffusion involves calculating a linear matrix-vector equation for each image row independently, as follows:

$$(E - 2\tau \cdot A_{row,i}^k)(L_i^{k+1}) = (L_i^k), \quad i=1,2,3\dots m \quad (19)$$

where L_i^k and L_i^{k+1} , respectively, denote the i th row vectors of the smoothed image at the k th and $k+1$ th iterations. In practice, they are transposed before the calculation. E is the unit matrix. $A_{row,i}^k$ is the diffusive matrix operator for the i th row and is given as

$$A_{row,i}^k = \begin{bmatrix} a_1^i & b_1^i & 0 & \dots & 0 \\ c_2^i & a_2^i & b_2^i & 0 & \dots \\ 0 & \dots & \dots & \dots & 0 \\ \dots & 0 & c_{n-1}^i & a_{n-1}^i & b_{n-1}^i \\ \dots & 0 & \dots & c_n^i & a_n^i \end{bmatrix} \quad (20)$$

where

$$\begin{cases} a_l^i = \begin{cases} -(g_{i,1}^k + g_{i,2}^k)/2, & l = 1 \\ -(g_{i,l+1}^k + g_{i,l-1}^k + 2g_{i,l}^k)/2, & l = 2, \dots, n-1 \\ -(g_{i,n-1}^k + g_{i,n}^k)/2, & l = n \end{cases} \\ b_l^i = (g_{i,l+1}^k + g_{i,l}^k)/2, & l = 1, \dots, n-1 \\ c_l^i = (g_{i,l-1}^k + g_{i,l}^k)/2, & l = 2, \dots, n \end{cases} \quad (21)$$

Here, $g_{i,l}^k$ is the diffusivity at pixel (i, l) . It can be seen that the operator $A_{row,i}^k$ is a tridiagonal matrix, and thus, $(E - 2\tau \cdot A_{row,i}^k)$ is also tridiagonal. Then, equation (19) can be quickly solved using the Thomas algorithm, whose time complexity is linear to the image pixel number. One can refer to [26] to see the detailed procedure. After the row diffusion, each row vector is recombined to form the row-diffused image L_x^{k+1} .

2. Vertical diffusion.

The vertical diffusion is similarly calculated as

$$(E - 2\tau \cdot A_{col,j}^k)L_j^{k+1} = L_j^k, \quad j=1,2,3\dots n \quad (22)$$

where L_j^k , L_j^{k+1} are, respectively, the j th column vectors of the smoothed image at the k th and $k+1$ th iterations. The column diffusive matrix operator $A_{col,j}^k$ is given as

$$A_{col,j}^k = \begin{bmatrix} a_1^j & b_1^j & 0 & \dots \\ c_2^j & a_2^j & b_2^j & 0 & \dots \\ 0 & \dots & \dots & \dots & 0 \\ \dots & 0 & c_{m-1}^j & a_{m-1}^j & b_{m-1}^j \\ \dots & 0 & c_m^j & a_m^j & \dots \end{bmatrix} \quad (23)$$

where

$$\begin{cases} a_l^i = \begin{cases} -(g_{1,j}^k + g_{2,j}^k)/2, & l = 1 \\ -(g_{l+1,j}^k + g_{l-1,j}^k + 2g_{l,j}^k)/2, & l = 2, \dots, m-1 \\ -(g_{m-1,j}^k + g_{m,j}^k)/2, & l = m \end{cases} \\ b_l^i = (g_{l+1,j}^k + g_{l,j}^k)/2, & l = 1, \dots, m-1 \\ c_l^i = (g_{l-1,j}^k + g_{l,j}^k)/2, & l = 2, \dots, m \end{cases} \quad (24)$$

Similarly, equation (22) is solved via the Thomas algorithm. The diffused column vectors are then combined to form the column-diffused image L_y^{k+1} .

3. Averaging.

Last, the smoothed image of the $k+1$ th iteration is obtained by averaging L_x^{k+1} and L_y^{k+1} :

$$L^{k+1} = (L_x^{k+1} + L_y^{k+1})/2 \quad (25)$$

The time step τ in the AOS scheme is usually set to 5 to achieve a tradeoff between the speed and accuracy [26]. For the setting in the proposed illumination estimation, there is no strict demand for accuracy because coarse smoothness is tolerated as long as the boundary edges are preserved. τ is set to 15 in this paper.

2) Multi-resolution Technique

Multi-resolution processing is an effective way to reduce the computation complexity because the computation is focused on a lower resolution that has a smaller data size yet remains the main structure of the original image. We simply decimate the image by a factor of 2 repeatedly to form the multi-resolution pyramid hierarchy, then start estimating the illumination at the coarsest resolution and expand the result by interpolation to be the initial image for the next resolution. To prevent excessive loss of the boundary information, the number N of downsampling is set in such a way that the size of the coarsest resolution is no smaller than 200:

$$N = \text{floor} \left[\log_2 \left(\frac{\max(rn, cn)}{200} \right) \right] \quad (26)$$

where rn and cn are the sizes of the rows and columns, respectively, and $\text{floor}[\cdot]$ is the down integral function.

It should be noted that the boundaries can be blurred and shifted during each interpolation. Thus, a restoration measure is taken to address the problem:

$$L_n = (1 - w) \cdot L_{\uparrow n+1} + w \cdot S_n \quad (27)$$

where

$$w = \frac{B_{\uparrow n+1}}{\max(B_{\uparrow n+1})} \quad (28)$$

where L_n is the image to be initialized at the n th layer.

L_n is obtained by taking the weighted sum of the interpolated illumination $L_{\uparrow n+1}$ at the end of the coarser $n + 1$ th layer and the original image S_n at the n th layer. The weight w is calculated by normalizing the boundary template $B_{\uparrow n+1}$ of $L_{\uparrow n+1}$. It can be seen that w is large in the boundary areas, and thus, the sharp boundary edges in S_n replace the blurred counterparts in $L_{\uparrow n+1}$.

In applications that utilize both the AOS scheme and multi-resolution technique, the number of iterations deployed in each resolution is settled based on two principles: 1) The total number of iterations is 10. 2) The finest resolution (original image) has 0 iterations. The remaining levels except for the coarsest level have 1 iteration, and the coarsest level has the remaining iterations. The motivation for this setting is that with more resolution levels (for larger images), there is less of a need to perform diffusion because the multiple downsamplings already have a blurring effect. The experiments in Section IV-B will illustrate that this strategy is sufficient for contrast enhancement.

E. Discussion about Well-posedness

As mentioned earlier, there is potential concern for ill-posedness of the nonlinear diffusion. In theory, the ill-posedness causes both forward and backward diffusions in the iterative process, which influences its convergence to a globally constant solution [22]-[24]. For our task, it is necessary to guarantee a sole evolving direction and an information-reducing process for illumination estimation.

Fortunately, it is sufficient to ensure well-posedness by imposing implicit regularizing effects on the discretization scheme of the equation. As was verified in [26], for a discretization procedure as follows

$$S^{k+1} = Q(S^k) \cdot S^k \quad (29)$$

if $Q(\cdot)$ has such properties as continuity, nonnegativity, symmetry, unit row sum, positive diagonal elements and irreducibility, then the diffusion filtering creates a smoothing scale space that guarantees convergence to a constant solution as the number of iterations tends to infinity. The former two conditions are inherited directly from the conductance function, whereas the latter four are related to the intrinsic structure of the discretization scheme. Because it was proven in [26] that the explicit scheme (12), semi-implicit scheme (17) and AOS scheme (18) meet the latter four conditions unconditionally, we must verify only the conductance function. It can be easily checked that the conductance function (9) adopted in the paper is continuous and nonnegative, and therefore, we guarantee well-posedness for our diffusion process.

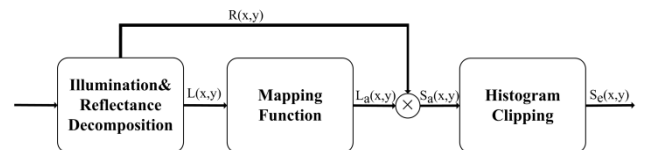


Fig. 3. Flow chart of the proposed enhancement algorithm.

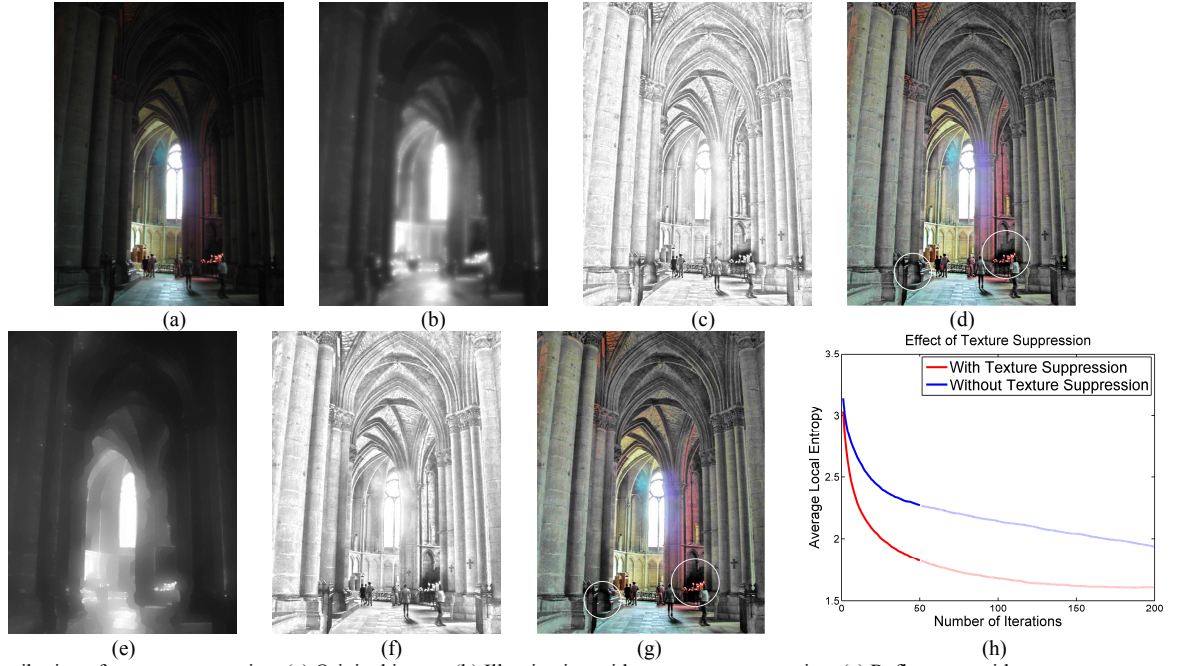


Fig. 4. Contribution of texture suppression. (a) Original image. (b) Illumination without texture suppression. (c) Reflectance without texture suppression. (d) Enhanced image without texture suppression. (e) Illumination with texture suppression. (f) Reflectance with texture suppression. (g) Enhanced image with texture suppression. (h) Curves of average local entropy of two illuminations

III. PROPOSED CONTRAST ENHANCEMENT ALGORITHM

The procedures of the proposed algorithm mainly involve illumination estimation, illumination-reflectance decomposition, illumination adjustment, and recombination of the two components. This type of structure addresses the illumination problem effectively and is widely adopted in variational Retinex algorithms [14]-[17]. Note that our algorithm generally differs in two aspects: 1) the method for illumination estimation is via nonlinear diffusion; and 2) the mapping function for illumination adjustment is a logarithmic function.

Considering that placing the enhancement on the three channels of the RGB color space potentially causes a “graying out” effect [37], the enhancement process should take place in the luminance component of the image to preserve the chrominance. In the proposed algorithm, a color space conversion from RGB to HSV is performed at the beginning and a reverse conversion at the end.

Fig. 3 presents the procedure of the core enhancement process for the luminance component (i.e., the V component in HSV) of the image. In the decomposition process, the illumination component $L(x, y)$ of the image luminance $S(x, y)$ is first estimated using the nonlinear diffusion method provided in Section II-C, and then, the reflectance image $R(x, y)$ is obtained by dividing $S(x, y)$ by $L(x, y)$:

$$R(x, y) = \frac{S(x, y)}{L(x, y) + \delta} \quad (30)$$

where δ is a small value that avoids division by 0. As the textures are smoothed out in the illumination, they are stored in reflectance via the division. This step is crucial for the final local contrast enhancement.

Then, the illumination image $L(x, y)$ is adjusted by a mapping function. Because the logarithmic function is strongly correlated to the nonlinear response of the human

eye to the luminance, the adjustment is given as

$$L_a(x, y) = \log_2(L(x, y) + 1) \quad (31)$$

In practice, the $\log$ function has strong compression while preserving the brightness order, which makes the enhanced image look natural. This step is important for elevating the global brightness.

The adjusted $L_a(x, y)$ is then multiplied by the reflectance image $R(x, y)$ again to obtain the adjusted luminance $S_a(x, y)$:

$$S_a(x, y) = R(x, y) \cdot L_a(x, y) \quad (32)$$

Next is the histogram clipping step, which cuts 0.5% of the pixels at the two ends of the histogram. This process can both remove the influence of a few pixels that have extreme luminance values and slightly enhance the image contrast. The last step is to rescale the dynamic range to [0 255] to obtain the enhanced luminance $S_e(x, y)$.

IV. EXPERIMENTAL RESULTS AND DISCUSSION

This section presents several discussions and experiments to verify the properties and the performance of the proposed algorithm. For better presentational accuracy, the experiments in Section IV-A are accelerated by only the AOS scheme with the stopping criterion (14). For Section IV-B, which covers practical use, both the AOS scheme and multi-resolution technique are applied without the stopping criterion, to obtain higher computational efficiency. The sources of the degraded image used for our experiments are as follows: Figs. 4-5, 12 from the free database at IPOL Journal website, Figs. 7-9 from the NASA Retinex webpage, and Figs. 10-11 from the image library provided by the author of the reference [13].

A. Properties Analysis

First, we settle two parameters, namely, the Gaussian scale σ for surround suppression and the suppression



Fig. 5. Decomposition by the three methods. (a) Original image. (b) Illumination by robust envelop. (c) Illumination by bilateral filters. (d) Illumination by the proposed method. (e) Reflectance by robust envelop. (f) Reflectance by bilateral filter. (g) Reflectance by the proposed method. (h) Enhanced image by robust envelop. (i) Enhancement image by bilateral filters. (j) Enhanced image by the proposed algorithm.

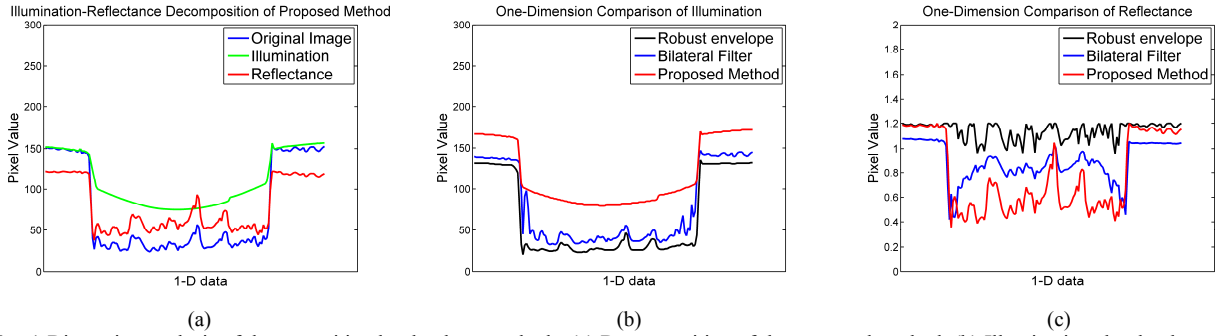


Fig. 6. 1-Dimension analysis of decomposition by the three methods. (a) Decomposition of the proposed method. (b) Illuminations by the three methods. (c) Reflectance by the three methods.

strength α , both of which impact the effects of the surround suppression. The variable σ indicates the radius of the surround with a specific amount of texture. The variable α simply determines the strength of the suppression. This parameter is important to the effect of the suppression because an excessively large α not only eliminates textures but also impairs some parts of the boundaries. On the other hand, texture suppression has little effect for small values of α . We refer to the analysis of the parameter combination in [35] and chose 1.2 for α . This value has a relatively large suppressive effect because we require quick effect within a few iterations. We test σ for dozens of images and find that although it trivially changes the local appearance of illumination, it scarcely affects the enhanced image. It has a relatively wide permissive range, within which there is no great divergence of enhanced

images. Despite this small robustness, it is reasonable to vary the area of the surround in direct proportion to the image area, which leads to

$$\sigma = c \cdot \sqrt{A} \quad (33)$$

where A is the image area, and c is a ratio coefficient, which is set to 0.0032 in this paper.

Next, we verify the influence of the entire texture suppression to the decomposition and enhanced image. Fig. 4 presents a comparison, where Fig. 4(a) is processed via the proposed algorithm into two groups: one is treated with texture suppression and the other without the suppression. This approach is equivalent to setting α to 1.2 and 0, respectively. As Fig. 4(b) and Fig. 4(e) show, the estimated illumination image without the suppression has such defects that the boundary areas are not well preserved

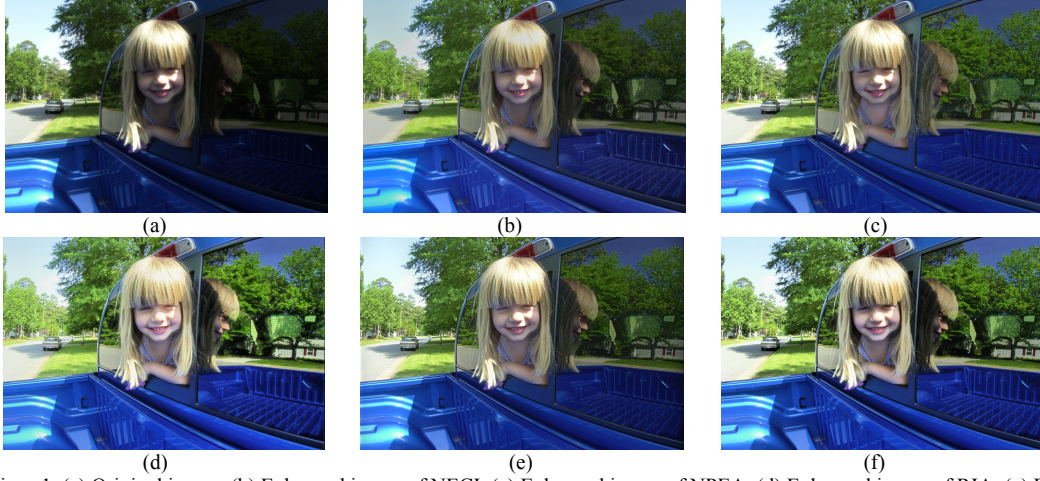


Fig. 7. Comparison 1. (a) Original image. (b) Enhanced image of NECI. (c) Enhanced image of NPEA. (d) Enhanced image of RIA. (e) Enhanced image of TVR. (f) Enhanced image of the proposed algorithm.



Fig. 8. Comparison 2. (a) Original image. (b) Enhanced image of NECI. (c) Enhanced image of NPEA. (d) Enhanced image of RIA. (e) Enhanced image of TVR. (f) Enhanced image of the proposed algorithm.

and that the texture areas gain inadequate smoothness. The illumination image with the suppression is much smoother in the texture areas inside the objects, while the boundaries are kept sharp. Comparing Fig. 4(c) with Fig. 4(f) shows that the reflectance image with the suppression has rich texture details and decent contrast, whereas the reflectance without the suppression appears slightly flat-looking. Because the enhanced image presents the features as reflectance, the result presented in Fig. 4(g) with the suppression has a greater rendition and larger contrast, especially in the areas that are enclosed by white circles. In Fig. 4(h), the average local entropy [38] is used to quantify the textural information reduction in illuminations with and without suppression along with iterations. The number of iterations is set to 200 for more information, and the iteration stopping point is actually around 50, as guided by the stopping criterion (14). For an image size of $m \times n$, the average local entropy is expressed as follows:

$$MLE = \frac{1}{mn} \sum_{i=1}^m \sum_{j=1}^n LE(i, j) \quad (34)$$

where $LE(i, j)$ denotes the entropy of a 9×9 window centered at (i, j) . As seen from Fig. 4(h), at the beginning, the red curve (with suppression) declines at a faster speed than the blue curve (without suppression), which implies that the suppression facilitates the vanishing of textural information. At the later period of the iterations, the red curve almost remains flat, whereas the blue curve is still at a slow linear decline. Therefore, the texture suppression mechanism contributes to the estimation of the expected illumination and avails the contrast enhancement.

Fig. 5 presents a comparison of the illumination-reflectance decomposition process among three enhancement algorithms, namely, Retinex with robust envelope [17] on behalf of the variational method, Retinex with bilateral filter [10] on behalf of the nonlinear filter method, and our proposed algorithm. The comparison reveals how illumination estimation affects the visual quality of the enhanced image. As seen from Fig. 5(b) and Fig. 5(c), the illuminations estimated by the former two methods are smooth at a very small scale. The

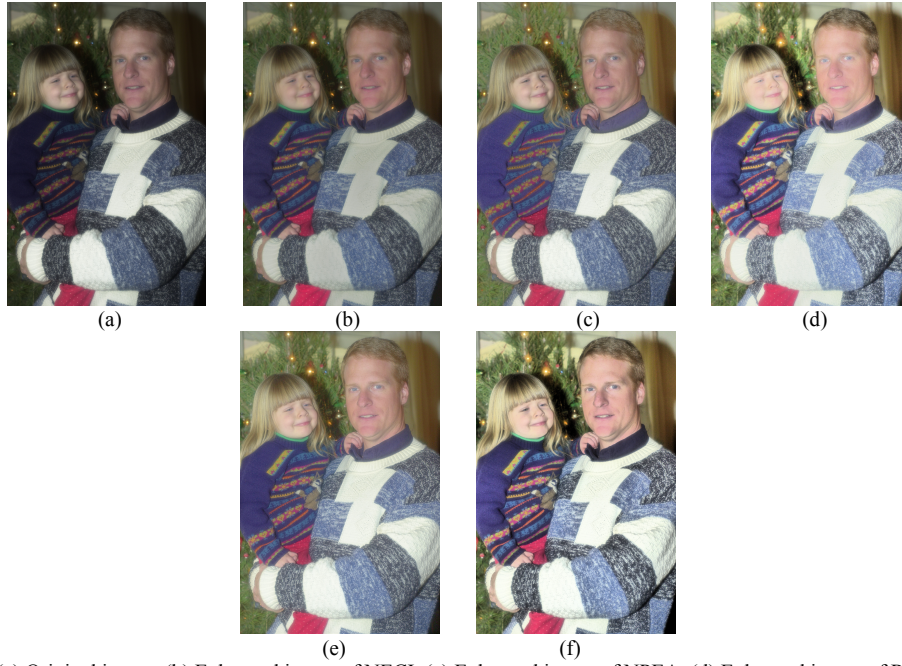


Fig. 9. Comparison 3. (a) Original image. (b) Enhanced image of NECI. (c) Enhanced image of NPEA. (d) Enhanced image of RIA. (e) Enhanced image of TVR. (f) Enhanced image of the proposed algorithm.

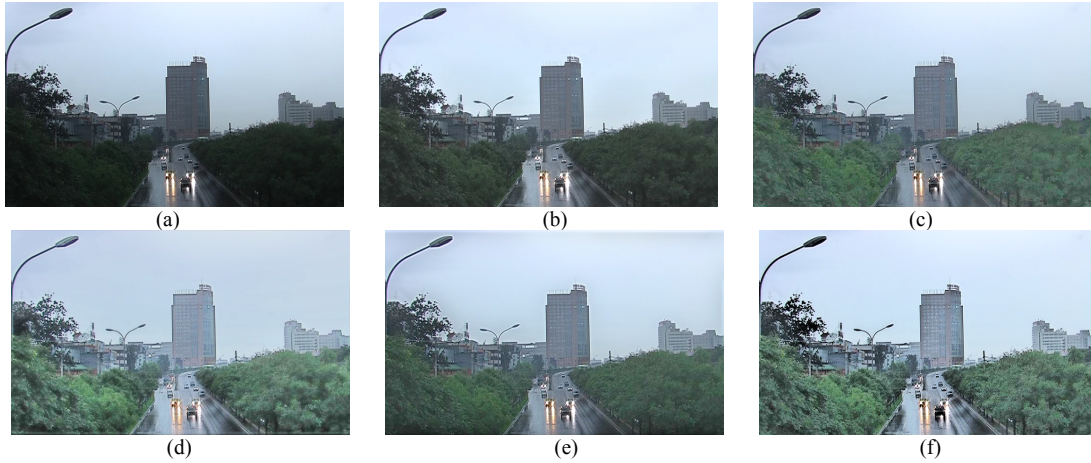


Fig. 10. Comparison 4. (a) Original image. (b) Enhanced image of NECI. (c) Enhanced image of NPEA. (d) Enhanced image of RIA. (e) Enhanced image of TVR. (f) Enhanced image of the proposed algorithm.

corresponding reflectance (Fig. 5(e) and Fig. 5(f)) and enhanced images (Fig. 5(h) and Fig. (i)) suffer from serious detail loss. In contrast, the proposed method has better performance in both the illumination estimation and contrast enhancement. Fig. 6 shows the one-dimension analysis of the three decompositions, where more specific details can be seen. The location of the extracted 1-D data corresponds to the white line in Fig. 5(a). As is shown clearly in Fig. 6(a), the illumination curve of the proposed algorithm changes abruptly only at the boundaries on the two sides and is as smooth as the regional average elsewhere. This circumstance is very close to the condition of the expected illumination: smooth as a constant inside regions and discontinuous in the region boundaries. This important feature of illumination facilitates the texture preservation in both the reflectance and enhanced image. As the comparison in Fig. 6(c) shows, the reflectance of the proposed method has higher contrast than the other two methods.

B. Comparison of Results

In Figs. 7-12, the proposed algorithm is compared with four recently proposed enhancement algorithms, namely,

natural color enhancement algorithm (NECI) [12], naturalness preserved enhancement algorithm (NPEA) [13], Retinex with illumination adjustment (RIA) [16], and Retinex by total variation (TVR) [15]. All these algorithms are based on illumination-reflectance decomposition. To be more specific, NECI and NPEA estimate illumination by means of convolutional filtering with nonlinear filters, whereas RIA and TVR are based on variational framework. Simulation materials of NPEA, RIA and TVR are the open MATLAB codes provided by the authors. As seen from Figs. 8-11, NECI tends to unduly preserve the image's black-white order and sacrifices the local contrast. Some of the images, such as Figs. 10-12(b), still suffer from illumination unevenness. NPEA is effective when addressing images of relatively small size, but it loses its merits as the image size grows. The main reason is that the size of its filter mask for the illumination estimation is limited, which results in a small scale of smoothness in the illumination. This deficiency leads to a flat-looking appearance of the enhanced image, especially in Fig. 9(c). RIA is effective in contrast enhancement, but it tends to unnecessarily soften the image due to its L_2 -based sparsity



Fig. 11. Comparison 5. (a) Original image. (b) Enhanced image of NECI. (c) Enhanced image of NPEA. (d) Enhanced image of RIA. (e) Enhanced image of TVR. (f) Enhanced image of the proposed algorithm.

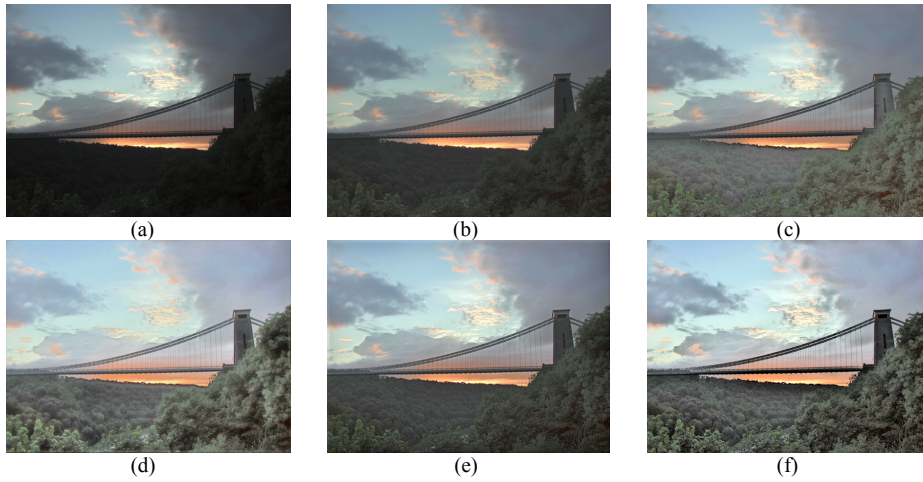


Fig. 12. Comparison 6. (a) Original image. (b) Enhanced image of NECI. (c) Enhanced image of NPEA. (d) Enhanced image of RIA. (e) Enhanced image of TVR. (f) Enhanced image of the proposed algorithm.

Table 1 Comparison of IEM

Methods Images	NECI	NPEA	RIA	TVR	Prop.
Fig. 7(a)	1.14	1.42	1.68	1.52	1.97
Fig. 8(a)	0.89	1.22	1.24	1.32	1.48
Fig. 9(a)	0.94	1.31	1.50	1.47	2.27
Fig. 10(a)	1.18	1.36	1.47	1.45	2.22
Fig. 11(a)	1.42	1.52	1.83	1.59	2.33
Fig. 12(a)	1.13	1.83	2.03	1.81	3.00

term imposed on the reflectance gradients. The edges of the enhanced images are not sharp, and some of the results are unsatisfactory in visual quality, as shown in Fig. 10(d) and Fig. 12(d). The TVR gains greater sharpness than RIA due to the TV term on the reflectance, but its capability of contrast improvement is unsatisfactory. In general, the proposed algorithm is more reliable and visually outperforms the other four algorithms.

In Table 1, we use an objective measure called Image Enhancement Metric (IEM) [39] to assess the performance of the above five algorithms. The IEM takes the ratio of local contrast in the enhanced image to that in the original image as the evaluation value. The higher the IEM value becomes, the better the improvements that are obtained in the contrast and sharpness [39]. Table 1 shows that the

evaluation values mostly coincide with the visual quality of the images. Because the proposed algorithm visually exceeds the others with regard to both the local contrast and texture sharpness, it has the highest evaluation values for all images.

The computational complexity of the core process of our algorithm is relatively low. Table 2 shows the main computational requirements of each step in a single diffusion iteration. We can see that because the AOS scheme has a linear time complexity [26], the dominant time-consuming step is the convolution for computing the suppression term in the texture suppression task. In practice, this process is implemented by transforming images into the frequency domain using radix-2 FFT (Fast Fourier Transform) and then performing multiplication. The result is obtained by another IFFT (Inverse Fast Fourier Transform). The time complexities of both the FFT and IFFT are $O(N \cdot \log_2 N)$, where N is the number of image pixels [40]. Outside the illumination estimation process, a complex procedure is the histogram clipping. Fortunately, it can be efficiently performed by building a cumulative histogram of the pixel values and then picking the corresponding quantiles from the two ends, which in total costs $O(N)$ [41]. To conclude, the proposed algorithm has a time complexity of $O(N \cdot \log_2 N)$.

The total process time of the proposed algorithm depends mostly on the number of iterations, which can be greatly reduced by the multi-resolution technique.

Table 2 Complexity of Single Diffusion Iteration

(N: number of pixels in image. M/D: multiplication/division. A/S: addition/subtraction)

Task	Procedure	M/D	A/S	Time Complexity
Texture Suppression	compute $ \nabla L(x,y) $	4N	3N	$O(N \cdot \log_2 N)$
	compute suppression term $t(x,y)$	$6N \cdot \log_2 N + 4N$	$6N \cdot \log_2 N + 2N$	
	compute boundary template $B(x,y)$	N	N	
AOS scheme [26]	create system matrix	2N	6N	$O(N)$
	Thomas algorithm	10N	6N	

Table 3 Comparison of Time Expenditure

Methods \ Images	Size	NPEA	RIA	TVR	Prop.(AOS+ Multi resolution)
Fig. 7(a)	656×1000	46.14	1.52	28.86	1.04
Fig. 8(a)	1000×656	46.06	1.50	23.78	0.96
Fig.9(a)	2000×1312	190.26	5.32	63.45	3.05
Fig. 10(a)	434×718	22.49	1.26	21.76	0.90
Fig. 11(a)	492×749	26.59	1.60	24.76	1.08
Fig. 12(a)	450×600	18.94	0.76	12.73	0.59

Considering the comparability of the simulation codes, we make a comparison in Table 3 with regard to the process time among NPEA, RIA, TVR and the proposed algorithm. The conditions for our comparison experiment are as follows: PC with Intel i3 2.20 GHz CPU, 6 G RAM, Windows 7, and MATLAB 2010b. Each result is obtained by running 10 simulations and taking the minimum value. As seen from Table 3, the proposed algorithm with dual accelerations has the lowest process time. One could notice that Fig. 7(a) and Fig. 8(a) have the same size but differ in process time when applying TVR. This circumstance occurs because the process time of TVR is determined by its stopping condition, which is relevant to the image structure rather than the size. It is also interesting to note that for our algorithm, there is a case in which the process time has an inverse correlation with the image size, as seen in the results for Fig. 8(a) and Fig. 11(a) in Table 3. This circumstance occurs mainly because according to the downsampling criterion (26), Fig. 8(a) requires more downsampling, and its coarsest level in which the majority of the diffusion process locates has a smaller size than that of Fig. 11(a). This effect enables the dual accelerations to gain some robustness in the process time because the main diffusion process lies in the coarsest layer.

V. CONCLUSIONS

This paper discusses the issue of illumination estimation in many enhancement algorithms that are based on illumination-reflectance decomposition. An assumption for the expected illumination that avails the contrast enhancement is also specified. A method based on nonlinear diffusion is proposed to realize the correct illumination estimation. This method solves the problem of

the limited contrast enhancement that results from the low smoothing capability of a traditional nonlinear filter. The analytical experiments show that the proposed enhancement algorithm achieves better visual quality than some previous algorithms. The acceleration methods enable the algorithm to have high computational efficiency. However, color management is not involved in this paper. In some special cases in which the scene is illuminated by a colored ambient light, balancing the image color is required. Because directly extending our algorithm to three color channels would cause a graying-out effect, more work should be performed in this area. This challenge will be our future work.

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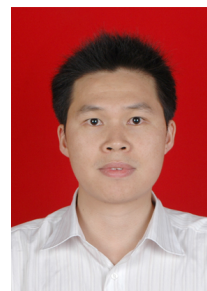
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